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Review Article

Aggregation of Correlates of Female Labour Force Participation

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Abstract

Studies on female labour force participation (FLFP) and economic development gave contrasting results, either supporting the existence of a U-shaped relationship or against such relationship. Interpretations of results of such studies need caution due to the associated problem areas and selection of indicators which are non-exhaustive or strongly correlated. Avoiding problems of logarithmic transformations, scaling/normalization and issues related to multiple regressions, the paper gives two countries specific FLPR indices at *t*-th year I_{FLPR_t} considering all relevant indicators with different score-ranges, distributions and inter-correlations. The index I_{FLPR_t} by multiplicative aggregation (Method-1) and cosine similarity (Method-2) are linearly related and satisfy desirable properties like monotonically increasing continuous scores, time-reversal test, formation of chain-indices, facilitate identification of critical dimensions or indicators, measurement of progress across time, etc. Considering theoretical advantages and applications, I_{FLPR_t} by Method-1 is recommended for easy comprehension. Future studies on empirical investigation of the properties of the proposed measures of I_{FLPR_t} suggested.

Classification Codes: O10; O52; J21

Keywords: female labor force participation; labour markets; gender gaps; regression, geometric mean; cosine similarity

Introduction

Female labour force participation (FLFP) is taken as number of female labor participants of age 15–64 divided by the total female population in the same age group, where labor force participation includes those employed plus unemployed (actively seeking work). FLFP has

two major implications:(i) women's empowerment to promote equal economic rights, access to employment, and economic activities, and control over economic resources across gender and (ii) gender inequality to achieve the targets of Goal-5 of Sustainable Development Goals (SDG-5) by 2030 which includes among others recognition of contributions of unpaid and domestic work, equal opportunities, participation in education and employment [11].

However, gender gaps exist in health, education, policy areas, and economic participations as per Global Gender Gap Report 2020 [32]. Globally, FLFP is little over 50%, against 80% for men and gender gap in participation is highly significant in South Asia, Middle East and North Africa where the participation rate of men exceeds three times the rate among females [16]. Non-utilisation of women to reach their full potential signifies tremendous loss of human capital and even a roadblock to economic advancement [24]. FLFP is an important dimension of Gender Inequality Index (GII) by (UNDP, 2022) [29], Gender Gap Index (GGI) [32].

The U-shaped female labour force function (FLFF) curve based on income effect (shift from traditional farm activities to works in secondary/tertiary sector) and substitution effect (rise in educational level) results in increased value of women's time in the agriculture dominated economy and encourages women to move back into the paid labour force with rise in service or tertiary sector [14]. FLFF curve is negatively slopped initially with industry dominated economic development, followed by plateau and increasing trend with increase in level of education of women resulting in increased value of women's time in the market giving the U-shape. However, the relationship is influenced by a host of factors like availability of job opportunities, socio-geo-economic pattern of living which prevents the females to move to other places in search of jobs; different allocations of time and efforts by gender in paid and unpaid works, policies and legislation of national governments, etc.

Empirical investigations on U-shaped FLFF curve have given contrasting results. In the context of India, (Olsen and Mehta, 2006) [20], found U-curve between employment and female educational status. Women of poor families work both at home and out of home. But when their income levels improve, they leave their outside works and concentrate on their household activities. Well educated women of higher income groups employ domestic helps and concentrate more on their economic activities out of their homes. Inverted U-shaped curve was observed between FLFPR and income with inflexion point at extremely high-income levels [23], between literacy rate and FLFPR at Uttarakhand state of India [1], and the Goldin hypothesis did not hold true for rural areas. Dispute exists regarding verification of the U-shaped feminizing theory [3]. Need is felt for consideration of issues and methods of finding relationship between estimates of FLFPR and its correlates. Methodological issues in empirical relationships of FLFPR with

its correlates by multiple linear regressions involving number of countries, transformations of the chosen variables, etc. might have resulted in divergent results.

The paper gives two methods of finding multidimensional index of FLPR of a country at *t*-th year I_{FLPR_t} by aggregating all chosen correlates of FLPR facilitating better comparisons, plotting its fluctuations across time and statistical test of significance.

Literature survey

Estimation of FLPR depends heavily on the data, methods of estimation and may not support the U-shaped hypothesis for non-OECD countries [13]. In India, FLFPR declined from 34.1% in 1999-00 to 27.2 % in 2011-12, despite strong economic growth associated with rising wages and incomes, unlike urban women for whom FLFPR increased from 14.6% to 15.5%. Decreasing trend of FLFPR was also observed for rural women. For example, FLFPR in Bihar declined between 2004–05 and 2018–19, with a modest increase after 2018–19, despite continuous economic growth rates [21]. However, women working in home were counted as unpaid workers and not counted in in FLFPR in 2011-12. As per ILO estimates, FLFPR in India was 23.5% in 2019 which has improved to 37.0% as per the Periodic Labour Force Survey Report 2022-23, released by the Ministry of Statistics and Programme Implementation, Govt. of India on 9th October, 2023. The increased FLFPR using the usual definition of labour force (employed for at least 30 days in a year) signifies a considerable improvement towards women's empowerment and their active involvement in India's socio-economic and political development. The upturn in FLFPR could be attributable to Government policies and legislations including substantial initiatives targeting girls' education, skill development, entrepreneurship facilitation, safety in the workplace etc. and have played important roles in advancing the agenda of 'women-led development'. Growth of service sector contributing 53% India's GDP (2021-22) and generating large-scale employment of educated women has also contributed to improve FLFPR in India. *As per* the Economic Survey 2022-23, Ministry of Finance & Corporate Affairs, GoI, 2023, growth of the service sector was 8.4% (YoY) in FY 2022 and likely growth of 9.1% in FY2023 for the Gross Value Added (GVA) in the services sector.

Problem areas - Multiple Linear Regressions:

Data:

Measurements of factors influencing FLFPR are not uniform across countries and time. Women working in home where as unpaid workers were not counted in India's FLFPR in 2011-12. Usual definition of labour force considering those employed for at least 30 days in a year was adopted subsequently. National Sample Survey Organization (NSSO) used well defined activity status codes to each household member reflecting types of activities undertaken. For example, activity code 93 is assigned to domestic duties including free collection of goods (vegetable, firewood, cattle feed, etc.), tailoring, etc. for household use.

Use of a single self-reporting question in survey to measure labour force status of an individual, especially for rural population is prone to errors. This is against ILO recommendations of additional 'recovery questions' in the questionnaire.

Model:

Empirical investigations on FLPR often involve fitting regression equation. For example, [25] considered equation of the form.

$$FLPR_{it} = \alpha + \beta_1 \log(GDP_{it}) + \beta_2 \log (GDP_{it})^2 + error_{it} \quad (1)$$

The equation implies $\frac{\partial(FLPR)}{\partial \log (GDP)} = \beta_1 + 2\beta_2 \log(GDP)$ indicating that an increase in $\log(GDP)$ will have "stronger" impact in FLPR for more extreme values. Interpretation of negative value of α is difficult. Statistically significant negative value of β_1 and positive value of β_2 supports the U-shaped relationship, but does not help much the policy makers in deciding specific action to improve FLPR. $\beta_1 > 0$ and $\beta_2 > 0$ indicate positive transitions; $\beta_1 < 0$ and $\beta_2 < 0$ give negative transitions and $\beta_1 > 0$ and $\beta_2 < 0$ give rise to inverted U-shaped curve.

Highly correlated $\log(GDP)$ and $\log (GDP)^2$ gives rise to multicollinearity and may inflate the β −coefficient of the log GDP. Such regression equation does not account for country-specific effects because data is essentially "cross-sectional" and is subjected to unobserved heterogeneity bias. In general, a small negative partial regression coefficient for an independent variable (say X_2) could be negative, even though the scatter plot of Y on X_2 alone shows a positive relationship. The contradiction is due to strong positive relationship between two predictor variables X_1 and X_2 [15]. Within-group estimation may account for fixed effects by estimating the regression in first difference rather than in level [25]. However, within-group estimation could be biased if FLPR is persistent (because of correlation of residuals with the lagged dependent variable in the differenced data) and endogenous regressors.

Various trends in FLFPR in different countries at different time periods give rise to a number of puzzles and different conclusions [9]. The authors found weak correlation of FLFPR with log ($GDP_{Per\ capita}$), insignificant correlation with female gross enrolment ratio in secondary school; and positive correlation with proportion of women in parliament.

Selection of independent variables:

Altuzarra et al. (2019) considered log of GDP based on purchasing power of parity ($lnGDP_{pc_{it}}$), instead of $\log(GDP)$ and a set of additional variables (called control variables) like fertility rate, life expectancy, unemployment rate, secondary and tertiary education and found that U-shaped hypothesis holds if the coefficient of $lnGDP_{pc_{it}} < 0$ and the same for $ln(GDP_{pc_{it}})^2 > 0$ [3]. However, shape of FLPR – $lnGDP_{PC}$ relationship for a selected time-period may be different for different countries (or groups of countries) since such countries could be transiting on different parts of the U-curve.

Major factors other than GDP influencing FLPR are: level of education of women (Vlasblom & Schippers, 2004) [31], unemployment rate, urbanization offering more employment opportunities and more liberal socio-cultural attitudes (Tsani et al. 2023) [28], sectoral structure (Verme, 2015) [30], wages, social and cultural norms (Mehrotra and Parida, 2017; Dildar, 2015) [10,19], etc. However, fertility rate and unemployment rate are ambiguous since they could also be taken as outputs of higher FLPR [12,14]. Women education and fertility rates having high negative correlation may result in a collinearity problem [3]. Literacy rates may not adequately represent education levels to be employed in secondary or tertiary sectors.

Other factors influencing FLPR include:

- Immigration increasing supply of labor with possible decline in wages. Theoretical framework by Borjas (1995) showed impact of immigration in redistribution of wealth in an economy where the natives compete with immigrant workers in the labor market with reduced availability of job opportunities [5].
- Various welfare schemes like monthly payment to adult women which act as disincentives to FLPR.
- Religion and religious orthodoxy discouraging women to participate in labour force
- Environmental factors, macroeconomic policies specific to a country
- Socio-geo-economic pattern of living including legal and tax regulations, *openness* of the country, etc.

Selection of indicators and domains may be made to give a fair summary of FLPR and decide sound aggregation procedure accommodating all the chosen indicators with different score-ranges, distributions and inter-correlations. Possible aggregation procedures avoiding transformation could be geometric mean (GM) of vectors $\boldsymbol{X_c}$ and $\boldsymbol{X_0}$ representing the current and base period respectively or similarity between such vectors.

Correlations:

Correlates of FLPR having different distributions, units and score ranges may reflect different contributions of the correlates to FLPR. The assumption of linear relationship of each independent variable with FLPR (as the dependent variable) may not be satisfied by high value of r_{FLPR,X_i} since $|r_{X,Y}| \geq 0.9$ may not always confirm linearity between X and Y [6]. The author also gave example of change in value of r_{XY} (and direction) with change in score range for *X* following *N (0, 1)* and $Y = \frac{1}{\sqrt{2\pi}}\, e^{\frac{-1}{2}X^2}$ where r_{XY}= - 0.93 for $0 \leq X \leq 3.9$ and r_{XY}= 0.00036 for$-3.9 \leq X \leq 3.9$. In other words, data homogeneity may distort correlation coefficient. Non-verification of linearity between FLPR (as *Y*) and *i*-th independent variable (X_i) by constant value of $\frac{Y_i - Y_{i+1}}{X_i - X_{i+1}}$ for all values of X_i or by checking distribution of residual as normal with mean = 0 and constant variance and transferring each X_i to a common score range may raise question regarding validity of the multiple regression equation. Thus, relevance of independent variables goes beyond the observed correlations.

Transformations:

Logarithmic transformation of a variable (*X*) may change value and even direction of correlation between *X* another variable (*Y*). For example, Kovacevic, (2011) found $r_{Life\ expetancy,HDI} > r_{Life\ expetancy,GDP}$ but $r_{\ln(Life\ expetancy),HDI} < r_{\ln(Life\ expetancy),GDP}$ [17]. Logarithmic transformation fails to satisfy desired properties like Translation Invariance and consistency in aggregation [7].

Relative importances of the independent variables are given by β −coefficients in (1) when the variables are standardized. However, such standardization may not be possible if the purpose is to find FLPR for a single country for a given year.

Sample size:

Consideration of few countries (< 30) and observations on few time-periods may not satisfy the minimum sample size required to obtain reliable estimates of population parameters, for which usual power analysis is insufficient (Trafimow, et al. 2020) [27], since power analysis depends on the sample size and also on the expected effect size too [26]. Estimating FLPR of a country avoiding calculation and decomposition of correlation ratio by product of ratios of current and base period values of chosen independent variables may be desirable [4].

Path of improvement:

FLPR of a country is influenced by the policy decisions and how the resources are used. FLPR by multiple linear regressions fail to assess overall progress or decline registered by a country from the base period and drawing the path of the FLPR on year-to year basis. It would be desirable to find country specific FLPR on successive years by a method facilitating formation of chain indices i.e. $FLPR_{20} = FLPR_{21} \times FLPR_{10}$ from the zero-th time period (base period) with time periods marked as 1, 2, 3, so on.

Interpretations of results of the relationship between FLFP through regression need caution due to the associated problem areas, consideration of small time periods in analysis and non-exhaustive or strongly correlated selection of independent variables.

Proposed method:

Set up:

Let $X_{1t}, X_{2t}, \ldots\ldots, X_{nt}$ are the raw scores at *t*-th year of *n*-chosen indicators influencing FLPR of a country. Let values of the corresponding indicators at the base period are $X_{10}, X_{20}, \ldots\ldots, X_{n0}$.

As a part of pre-processing of data, (i) ensure that higher value of each indicator increases FLPR. For example, for the indicators where lower value tends to increase FLPR, reciprocal of such indicators may br considered, (ii) For indicators in ordinal scale, like attitudes, awareness, social rigidity discouraging women to work outside home, etc. obtained from survey using *K*-point scales (*K*= 2,3, ……,), convert each discrete raw scores to monotonically increasing, equidistant scores following normal distribution say $N(35, 10^2)$ to attain comparable results by the method suggested by [8].

Avoiding logarithmic transformations, scaling or normalization, weights and considering all relevant chosen indicators, irrespective of their inter-correlations, score-ranges Index of FLPR of a country at *t*-th year I_{FLPR_t} are proposed as:

Method-1: GM of ratios of X_{it} and X_{i0} i.e.

$$I_{FLPR_t} = \sqrt[n]{\frac{X_{1t}.X_{2t}.\ldots\ldots\ldots X_{nt}}{X_{10}.X_{20}.\ldots. X_{n0}}} \quad (2)$$

or equivalently by $I_{FLPR_t} = \prod_{i=1}^{n} \frac{X_{it}}{X_{i0}}$ (3)

Method-2: Cosine similarity between the vectors $\boldsymbol{X_t} = (X_{1t}, X_{2t}, \ldots\ldots\ldots, X_{nt})^T$ and $\boldsymbol{X_0} = (X_{10}, X_{20}, \ldots\ldots\ldots, X_{n0})^T$ making an angle θ between them i.e.

$$I_{FLPR_t} = Cos\theta = \frac{{X_t}^T X_0}{\|X_t\|\|X_0\|} \quad (4)$$

where length of the vector $\boldsymbol{X_0}$ is computed as $\|X_0\| = \sum_{i=1}^{n} {X_{i0}}^2$ and $\|X_t\| = \sum_{i=1}^{n} {X_{it}}^2$

$0 \leq Cos\theta \leq 1$. Lower $\theta \Longrightarrow$ higher $Cos\theta$ and lower $Cos\theta \Longrightarrow$ more homogeneous data. $Cos\theta_{ij} = \frac{{X_i}^T X_j}{\|X_i\|\|X_j\|}$ for $i \neq j$ reflecting association of *i*-th and *j*-th country ($i \neq j$) may be used for classification of countries.

Each of the equation (3) and (4) indicates overall socio-economic achievement of the country at the *t*-th year over the base period.

Assessment of improvement/decline:

Equation (3) helps to find improvement of FLPR by a country in two successive years by $I_{FLPR_t} - I_{FLPR_{(t-1)}} > 0$ or by $\frac{I_{FLPR_t}}{I_{FLPR_{(t-1)}}} > 1$. Improvement of the *i*-th indicator at *t*-th year over the base period is reflected by$\frac{X_{it}}{X_{i0}} > 1$. The *i*-th indicator is critical if $\frac{X_{it}}{X_{i(t-1)}} < 1$ and merits managerial attention for initiation of necessary corrective action.

For Method-1, $I_{FLPR_{t0}} * I_{FLPR_{0t}} = 1$. Thus, time-reversal test is satisfied. The index also satisfies $I_{FLPR_{20}} = I_{FLPR_{21}} * I_{FLPR_{10}}$enabling formation of chain indices and plotting I_{FLPR_t} graph of a country in successive years depicting improvement/decline since the base period. Two different countries may

be also be compared in terms of such graphs registered by the countries in longitudinal studies. If the base period data is replaced by the data of the previous year, I_{FLPR_t}will indicate growth on Y-Y basis.

For I_{FLPR_t}(Method-2), Chakrabartty, (2020) showed that: [8]

- $Cos\theta$ increases monotonically
- $\frac{Cos\theta_{i1}}{Cos\theta_{i0}}$ depicts progress or deterioration in year 1 from the base period
- Change of a country in successive years is evaluated by $\frac{Cos\theta_{ti}}{Cos\theta_{(t-1)i}}$
- Chain indices can be formed ensuring $\frac{Cos\theta_{i2}}{Cos\theta_{i0}} = \frac{Cos\theta_{i2}}{Cos\theta_{i1}} * \frac{Cos\theta_{i1}}{Cos\theta_{i0}}$

Relationship between I_{FLPR_t} as per two methods:

Taking log on both sides of (2), on I_{FLPR_t}(Method-1)

$\log I_{FLPR_t} = \sum_{i=1}^{n} logX_{it} - \sum_{i=1}^{n} logX_{i0} = \log [\frac{\|X_t\|}{\|X_0\|}]$

Since $\|X_t\| = \sqrt{\sum_{i=1}^{n} {X_{it}}^2} \Longrightarrow$

$log\|X_t\| = \frac{1}{2}[2logX_{1t} + 2logX_{2t} + \cdots \ldots \ldots \ldots + 2logX_{nt}] = \sum_{i=1}^{n} logX_{it}$

Thus, I_{FLPR_t}(Method-1) $= \frac{\|X_t\|}{\|X_0\|}$ (5)

Now, I_{FLPR_t}(Method-2) as $Cos\theta = \frac{\sum_{i=1}^{n} X_{it}X_{i0}}{\|X_t\|\|X_0\|} \Longrightarrow \frac{Cos\theta}{I_{FLPR_{t0}}(\text{Method 1})} = \frac{\sum_{i=1}^{n} X_{it}X_{i0}}{\|X_t\|^2}$

$\Longrightarrow Cos\theta = (\frac{{X_t}^T X_0}{{X_t}^T X})$. I_{FLPR_t}(Method-1)

Thus, I_{FLPR_t}(Method-2) $= (\frac{{X_t}^T X_0}{{X_t}^T X})$. I_{FLPR_t}(Method-1) (6)

Equation (6) confirms linear relationship of I_{FLPR_t} by Method-1 and Method-2. I_{FLPR_t} by each proposed method is simple to calculate, defined even if they are in percentages or skewed. $I_{FLPR_t}(Method-1) \times 100$ indicates percentage changes from the base period.

Mean and variance ofI_{FLPR_t}:

Mean and variance of $I_{FLPR_t}$for a group of countries may be found by considering logarithmic transformations since $\log(GM)$ approaches lognormal distribution [2]. Thus, mean and variance of $ln\, I_{FLPR_t}(\text{Method}-1)$ are $e^{\mu_X + \frac{\sigma_X^2}{2}}$and $e^{2\mu_X + \sigma_X{}^2}(e^{\sigma_X{}^2} - 1)$ respectively.

Mean and SD of $Cos\theta_i$ of *m*-countries for I_{FLPR_t}(Method-2), may be found by considering angles$\emptyset_1, \emptyset_2, \ldots \ldots \ldots, \emptyset_m$, each obtained for vectors of unit length [22]. Here, mean representing the most preferred direction is given by $\overline{\emptyset} = Cot^{-1}\frac{\sum_{i=1}^{m} Cos\emptyset_i}{\sum_{i=1}^{m} Sin\emptyset_i}$ and the dispersion by

$\sqrt{1-r^2}$ where $r^2 = (\frac{\sum Cos\emptyset_i}{m})^2 + (\frac{\sum Sin\emptyset_i}{m})^2$

The methods requires to convert $\boldsymbol{X_t}$and $\boldsymbol{X_0}$to $\boldsymbol{\pi_t}$and $\boldsymbol{\pi_0}$where $\pi_{it} = \sqrt{\frac{X_{it}}{\|X_t\|}}$ and $\pi_{i0} = \sqrt{\frac{X_{i0}}{\|X_0\|}}$ so that $\|\pi_t\|^2 = \|\pi_0\|^2 = 1$.

Thus, mean and SD of I_{FLPR_t}(Method 2) for *m*-countries respectively are:

Cos $(\bar{\theta}) = Cos(Cot^{-1}\frac{\sum Cos\theta_i}{\sum Sin\theta_i})$ and $\sqrt{1 - ([\frac{\sum Cos\emptyset_i}{m})^2 + (\frac{\sum Sin\emptyset_i}{m})^2]}$

Properties:

Following major desired properties of the proposed I_{FLPR_t} are satisfied by each of (3) and (4):

- Measures overall socio-economic improvement or decline of a country in the *t*-th year in comparison to the base year by a continuous variable and is independent of change of scale

- I_{FLPR_t} is monotonically increasing since increase in value of an indicator $(X_i) \Rightarrow$ increase in value of I_{FLPR_t}

- $\frac{Corresponding\ increase\ in\ I_{FLPR_t}}{Unit\ Increase\ in\ X_{it}}$ is constant, implying linearity between $I_{FLPR_t}$and X_{it}

- Significant reduction of trade-off among the indicators.

- Relative importance of *j*-th indicator may be assessed by $\frac{\nabla(I_{FLPR_t})}{\nabla X_j}$

- Ranking and classification of countries with respect to $I_{FLPR_t}$by Method-1 or Method-2.

- Not affected much by outliers. Linearly related I_{FLPR_t} by Method-1 and Method-2 produces no bias for economically developed or under-developed countries.

Applications:

$I_{FLPR_t}(\text{Method}-1)$ and $I_{FLPR_t}(\text{Method}-2)$ can be applied for data in percentages or skewed. Each method facilitates computation of the index for properly different sub-groups say rural or urban groups, soci-economically backward groups, educated or uneducated groups, etc.

The index I_{FLPR} may be correlated with actual FLPR to reflect association between them and regression equation can be fitted of FLPR on I_{FLPR} along with reporting of distribution of the residual.

In case, $r_{FLPR,I_{FLPR}}$ is not very high, the method suggested by Chakrabartty (2023b) transforming the variable Y to $\hat{\boldsymbol{y}} = \boldsymbol{G}.\|x\|\|y\|.x$ where G is the G-inverse of the matrix A = $\boldsymbol{x}.\boldsymbol{x}^{\boldsymbol{T}}$ resulting in perfect correlation i.e. $r_{X\hat{y}} = 1$ can be applied [6].

Limitations:

The index I_{FLPR} takes each indicator $X_{it} > 0$ for all values of $t = 0, 1, 2,\ldots\ldots$ and so on. If needed, zero target of an indicator like Gender inequality = 0 need to be modified as $\frac{Male}{Female} = 1$, failing which, a small value say 0.00001 may replace the zero target.

In case a new indicator is introduced, one needs to estimate its values in each year starting from the the base year. Assumes no missing data.

Discussion:

The proposed two indices of FLPR of a country at *t*-th year I_{FLPR_t} avoid problems of logarithmic transformations, scaling/normalization, finding weights and issues related to multiple regressions and consider all relevant chosen indicators, irrespective of their inter-correlations, score-ranges and distribution. The two indices of I_{FLPR_t} with no bias for developed or under-developed countries are linearly related and satisfy desirable properties like monotonically increasing continuous scores, satisfying time-reversal test and enables formation of chain indices and facilitate identification of critical indicators, measurement of progress of a country across time, etc.

I_{FLPR_t} facilitates ranking, comparison of countries for a given year or with respect to progress-paths registered by the countries since the base year. For a group of countries, it is possible to compute mean and SD of $I_{FLPR_t}(\text{Method}-1)$ and also $I_{FLPR_t}(\text{Method}-2)$.

Since the base period figures are different for different countries, comparison of countries may be meaningful in terms of progress made from base period or on Year-to-Year basis.

It is possible to undertake Statistical testing of (i) equality of mean of log (I_{FLPR_t}) of a pair of countries at a given year (ii) equality of mean of log (I_{FLPR_t}) of a country at two time periods.

Empirically, Chakrabartty (2021) found coefficient of variation (CV) for Method-2 was less than the same for Method-1c [7].

Conclusions:

Correlation of I_{FLPR_t} with FLPR will be same since I_{FLPR_t} by GM and Cosine similarity share a linear relationship. Each of the proposed measure of country specific I_{FLPR_t} with theoretical advantages and application to assess extent of improvement or deterioration over time, estimation of progress path and analysis under parametric set up will help the researchers and practitioners to draw meaningful conclusions. I_{FLPR_t} by GM approach is recommended for easy comprehension.

Numerical verifications of the properties of the proposed measures of I_{FLPR_t} may be undertaken to investigate relationships with actual FLPR data along with distribution of residuals for fitting regression equation of FLPR on I_{FLPR_t} or when I_{FLPR_t} is transformed by G-inverse to ensure perfect correlation.

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References

1. Agarwal, A. and Joshi, D. (2015). Goldin's 'U'- Shaped Curve Hypothesis on Female Labour Participation Rate: A Case Study of Uttarakhand. *International Journal of Development Research*, 5 (3), 3903 -3907
2. Alf, E.F. and Grossberg, J. M. (1979): The geometric mean: Confidence limits and significance tests, *Perception &Psychophysics*, 26 (5), 419-421
3. Altuzarra, A., Galvez- Galvez, C. and González Flores, A. (2019). Economic Development and Female Labour Force Participation: The Case of European Union Countries. *Sustainability*, 11, 1962; doi:10.3390/su11071962
4. Becker, SP, Burns, GL, Garner, AA, Jarrett, MA, Luebbe, AM, Epstein, JN., et al. (2018). Sluggish cognitive tempo in adults: psychometric validation of the adult concentration inventory. *Psychol. Assess.* 30, 296–310. doi: 10.1037/pas0000476
5. Borjas GJ (1995) The economic benefits from immigration. *J Econ Perspect* 9:3–22
6. Chakrabartty, S. N. (2023). Improving linearity in health science Investigations. *Health Science Journal*. Vol. 17 No. 4: 1010. DOI: 10.36648/1791-809X.17.4.1010
7. Chakrabartty, S. N. (2021).Wellbeing index avoiding scaling and weights. *Cultura, Educacióny Sociedad*, 12(2), 181-204.http://dx.doi.org/10.17981/culteduso.12.2.2021.11
8. Chakrabartty, Satyendra Nath (2020). Alternate Measure of Disability Intensity: Modified Rankin Scale. Journal of Stroke Medicine, 2(2) 111–118. DOI: 10.1177/2516608519870703
9. Chaudhary, R., & Verick, S. (2014). *Female labour force participation in India and beyond* (pp. 1-32). New Delhi: ILO.
10. Dildar, Y. (2015). Patriarchal Norms, Religion, and Female Labor Supply: Evidence from Turkey. *World Dev*. 76, 40–61.
11. Dugarova, E. (2018). Gender equality as an accelerator for achieving the Sustainable Development Goals. Discussion Paper. United Nations Entity for Gender Equality and the Empowerment of Women, New York, USA.
12. Fatima, A. and Sultana, H. (2009). Tracing out the U-shape relationship between female labor force participation rate and economic development for Pakistan. *Int. J. Soc. Econ.*, 36, 182–198.
13. Gaddis, I. and Klasen, S. (2014). Economic development, structural change, and women's labor force participation. *J. Popul. Econ*. 27, 639–681.
14. Goldin, C. (1994). The U - Shaped Female Labour Function in Economic Development and Economic History. In T. P. Schultz, *Investment in Women's Human Capital and Economic Development.* Chicago, USA: University of Chicago Press.
15. Hand, D.J. J. Douglas Carroll and Paul E. Green, with contributions by A. Chaturvedi, (1999). Mathematical Tools for Applied Multivariate Analysis, *J. of Classification* **16**, 312–314 (1999). https://doi.org/10.1007/s003579900060

16. International Labour Organization (ILO). (2022), Data retrieved from World Bank Gender Data Portal (SL.TLF.ACTI.ZS)
17. Kovacevic, M. (2011): *Review of HDI Critiques and Potential Improvements*, The Human Development Research Paper (HDRP) Series, Research Paper 2010/33
18. McKinsey Global Institute (2015). The Power of Parity: How advancing women's equality can add $12 trillion to global growth. *www.mckinsey.com/mgi.*
19. Mehrotra, S. and Parida, J.K. (2017). Why is the Labour Force Participation of Women declining in India? *World Dev*., 98, 360–380
20. Olsen, W and Mehta, S. (2006). A Pluralist Account of Labour Participation in India. Working Paper No GPRG-WPS-042. Economics Series Working Papers, University of Oxford, Department of Economics.
21. Pandey, Aviral (2023). Explaining the U-curve Trend of Female Labour Force Participation in Bihar. *EPW*, Vol. 58, Issue No. 30
22. Rao, CR. (1973): *Linear Statistical Inference and its Application.* 2nd Edition, Wiley Eastern Private Limited, New Delhi
23. Sai, PD and Kameswari, BL. (2022). The female labor force participation rate and India's economic development: tracing the U feminization hypothesis. *International Journal for Research Trends and Innovation*, Volume 7, Issue 8, 169 – 182.
24. Sanghi, S., Srija, A., & Vijay, S. S. (2015). Decline in rural female labour force participation in India: A relook into the causes. *Vikalpa*, 40(3), 255-268.
25. Tam, H. (2011). U-shaped female labour participation with economic development: Some panel data evidence, *Economics Letters*, Vol. 110, No. 2, 140–142.
26. Tong, T., Trafimow, D., Wang, T., Wang, C., Hu, L., & Chen, X. (2022). The A Priori Procedure (APP) for Estimating Regression Coefficients in Linear Models. *Methodology*, *18*(3), 203-220. https://doi.org/10.5964/meth.8245
27. Trafimow, D., Hyman, M. R., & Kostyk, A. (2020). The (im)precision of scholarly consumer behavior research. *Journal of Business Research*, *114*, 93–101. https://doi.org/10.1016/j.jbusres.2020.04.008
28. Tsani, S.; Paroussos, L.; Fragiadakis, C.; Charalambidis, I.; Capros, P. (2013). Female labour force participation and economic growth in the South Mediterranean countries. *Econ. Lett*., 120, 323–328.
29. UNDP (2022). Gender inequality index (GII) (http://hdr.undp.org/en/content/gender-inequality-index-gii).
30. Verme, P. (2015). Economic development and female labor participation in the Middle East and North Africa: a test of the U-shape hypothesis. *IZA J Labor Develop* **4**, 3. https://doi.org/10.1186/s40175-014-0025-z
31. Vlasblom, J.D. and Schippers, J.J. (2004). Increases in Female Labour Force Participation in Europe: Similarities and Differences. *Eur. J. Popul.* 20, 375–392.
32. World Economic Forum (2020). *Global Gender Gap Report 2020*. World Economic Forum, Switzerland. http://www3.weforum.org/docs/WEF_GGGR_2020.pdf